

• Addition

- Two matrices A and B must have the same dimensions
- Add the individual elements of the matrix

$$\circ \quad A + B = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{pmatrix}$$

• Multiplication

- Scalar: multiply each element of the matrix by the scalar quantity

$$\blacksquare \quad cA = \begin{pmatrix} ca_{11} & ca_{12} & \cdots & ca_{1n} \\ ca_{21} & ca_{22} & \cdots & ca_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ca_{m1} & ca_{m2} & \cdots & ca_{mn} \end{pmatrix}$$

- Matrix and vector multiplication

- In order for a matrix A and B to be multiplied, the number of columns of A must equal the numbers of rows of B .
- Each entry of AB at position (i, j) is the dot product of the i th row vector of A and the j th column vector of B .
 - i.e. $AB_{ij} = \vec{a}_i \cdot \vec{b}_j$
 - AB will have the dimensions of the number of rows of A by the number of columns of B .

$$\blacksquare \quad \text{Example: Let } A = \begin{pmatrix} 2 & 5 \\ -1 & 3 \\ 4 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 \\ -2 \end{pmatrix}. AB = \begin{pmatrix} -4 \\ -9 \\ 8 \end{pmatrix}$$

- The best way to not screw up is to write your product matrix in the middle with A on the left and B on top.
- $AB \neq BA$ (matrix multiplication is not commutative).

• Transposition

- The transposed matrix A^T is the matrix “reflected” across its diagonal.

$$\circ \quad \text{If } A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \text{ then } A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}$$

- If A is an $m \times n$ matrix, then A^T is an $n \times m$ matrix.
- A^T is also unique to A . The transpose of A^T is A .
- $(A + B)^T = A^T + B^T$ $(cA)^T = cA^T$
- $(AB)^T = B^T A^T$ $(A^T)^{-1} = (A^{-1})^T$